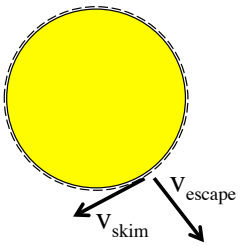
Problem 13.36

Relate the "skim speed" of an object moving in a circular path close to the earth's surface and the object's "escape velocity."



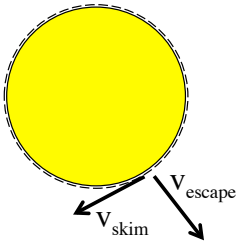
The *escape velocity* is the velocity an object at the earth's surface needs to break free of the earth's gravitational field (which is to say, the velocity required to get it to infinity) with no kinetic energy left over. That is:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}m\left(v_{escape}\right)^2 + \left(-\frac{GmM}{R_{earth}}\right) + 0 = 0 + 0$$

$$\Rightarrow \quad v_{escape} = \left(\frac{2GM}{R_{earth}}\right)^{1/2}$$

The *skim velocity* is the velocity an object would have to have at the earth's surface to travel in a circular path under a centripetal (gravitational) force. That is:



$$\underline{\sum F_{centripetal}:}$$

$$\frac{GmM}{\left(R_{earth}\right)^2} = ma_c = m\left(\frac{\left(v_{skim}\right)^2}{R_{earth}}\right)$$

$$\Rightarrow \quad v_{skim} = \left(\frac{GM}{R_{earth}}\right)^{1/2}$$

So:

$$\frac{v_{skim}}{v_{escape}} = \frac{\left(\frac{GM}{R_{earth}}\right)^{1/2}}{\left(\frac{2GM}{R_{earth}}\right)^{1/2}}$$

$$= \frac{1}{\sqrt{2}} = 1.42$$